Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Solids and Structures 42 (2005) 5758–5764

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

# Low energy impact evaluation using non conservative models

O. Jiménez [a,*], J.A. Sullcahuaman [b], M. Sánchez-Soto [c], A.B. Martinez [c]

[a] CIATEQ A.C. Calzada del Retablo 150, Col. FOVISSTE, C.P. 76150, Querétaro, Qro., Mexico
[b] Pontificia Universidad Católica del Perú, Sección Ingeniería Mecánica. Av. Universitaria cdra. 18 s/n, Lima 32, Peru
[c] Centre Catalá del Plástic, Universitat Politécnica de Catalunya. C/Colom 114, 08222 Terrassa, Spain

Received 9 February 2005
Available online 29 June 2005

## Abstract

The aim of the present work is the evaluation of mechanical properties of polymer and polymer matrix composites by using a low energy impact technique on a flexure plate configuration, which consists of a plate that is hit by a semi-spherical indenter. For the analysis of the force-time histories acquired experimentally a non elastic and non conservative model that includes the permanent deformations due the flexion and indentation produced by the impact is proposed. The model proposed has two systems set up in a serial arrangement. The first one is a spring-dashpot and the second is a hertzian spring-dashpot that simulates the indentation. Since the differential equation that describes this system does not have an analytical solution a 4th order Runge–Kutta algorithm was used. The overall energy loss was calculated by means of the restitution coefficient that was measured experimentally; these results were compared with those obtained solving the differential equation.

Eight sets of samples of polystyrene (PS) matrix composite with elastomeric and rigid dispersed phases were tested. A good correlation between the analytical and experimental results was observed, which allowed the calculation of the elastic modulus at high loading rates and the determination of the energy necessary to initiate damage of the specimen.


*Keywords:* Low energy impact; Composites; Plastics; Material evaluation; Non conservative model

## 1. Introduction

The instrumented impact tests on plastic and composite materials have been generalized as a consequence of the availability of more capable commercial equipments and the necessity of acquiring information of materials subject to a high speed loading.

* Corresponding author. Tel.: +52 442 1961500; fax: +52 442 2215243.
*E-mail address:* omar.jimenez@ciateq.mx (O. Jiménez).

0020-7683/$ - see front matter 
doi:10.1016/j.ijsolstr.2005.03.014

Previous works have been done (Martinez et al., 1994; Casiraghi et al., 1988; Pang, 1991) in order to increase the knowledge of this phenomenon and some methods have been developed to calculate the force history during the impact event. These methods that consider the material as an ideal elastic solid are based on spring models and low deformation theory, which implies that those form a conservative system without energy loss. However, the real systems are in some way non conservative (Martinez et al., 2001) and as a consequence, a complete model must take into account the sample flexion and indentation with their respective energy loss.

The goals of this work are, firstly, the development of a non conservative model that describes the real impact phenomenon taking into account the flexion and indentation components, and secondly, the evaluation of the model capacity to predict the force history or evaluate the material properties. In this order, samples of Polystyrene (PS), High Impact Polystyrene (HIPS) and their blends with and without glass beads have been tested.

## 2. Experimental details

The instrumented impacts test were carried out by means of a commercial equipment model DARTVIS by CEAST, a falling weight system was a ram of 0.744 kg and an indenter of 12.7 mm diameter, varying the impact energy by the change of the falling height. The impact tests were made at room temperature and the recorded signals were not filtered. The results had been analyzed using the methodology described by Martinez et al. (2001) and Jimenez et al. (2002), for the restitution coefficient and the mechanical properties, and the techniques described by Sullcahuaman (2001) for the impact velocity.

## 3. Materials

A commercial Lacqrene 1541 PS with 5–5.5% mineral oil as lubricant and a commercial Lacqrene 7240 HIPS (both from Elf-Atochem) were used. Also dilutions of 25, 50 and 75 wt.% HIPS in PS were prepared, adding to some blends 12.3 wt.% of Sovitec 050-20-010 glass beads (μeV) without surface treatment. With these materials 80 mm diameter and 4 mm width plates were made by injection molding to perform the impact test.

## 4. Model development

The flexion model proposed by Martinez et al. (2001) consists of a linear spring to represent the elastic behavior of the material and a linear dashpot that allows to take into account their energy loss. This model have a good success evaluating materials with a restitution coefficient near to 1, like PMMA and PS.

On the other hand, the indentation impact model proposed by Jimenez et al. (2002), is successful to evaluate the phenomena over a sample that is not allowed to be flexed, trough the use of a hertzian spring to represent the elastic component and a linear dashpot to represent the energy loss. In this and the model proposed by Martínez, the dashpot includes all the non elastic mechanisms.

Taking these models, a new one can be constructed involving the flexion and indentation components as shown in Fig. 1. This model is formed by the mass element $m$, two linear dashpots, one for indentation $C_i$ and other for flexion $C_f$, and two elastic elements, the first is the flexion linear spring $K_f$, and the second is the indentation hertzian spring $K_i$ (Rayleigh, 1906; Greszczuk, 1982). These elements behavior are described by Eqs. (1)–(5),

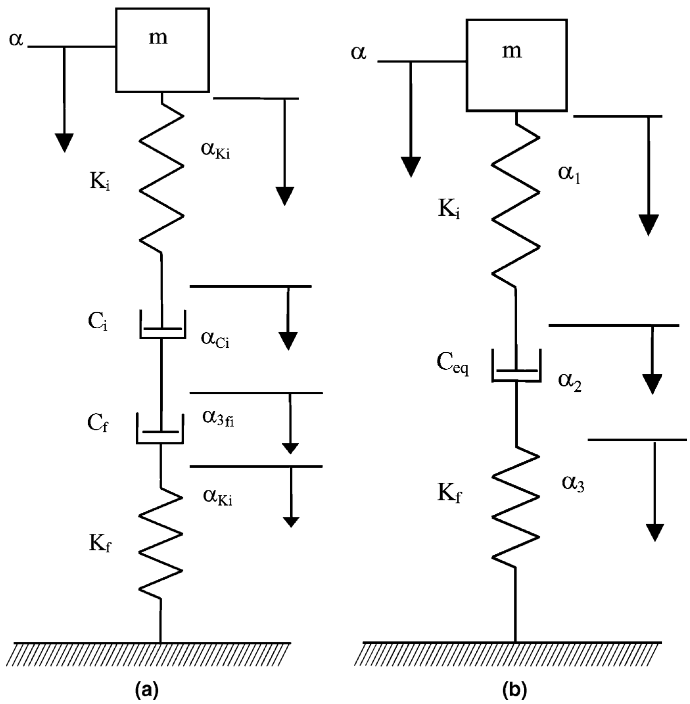


Fig. 1. Serial model flexion + indentation (a) complete; (b) simplified.

$$F_m = m\ddot{\alpha} + mg \tag{1}$$

$$F_{C_i} = C_i\dot{\alpha}_{C_i} \tag{2}$$

$$F_{C_f} = C_f\dot{\alpha}_{C_f} \tag{3}$$

$$F_{K_f} = K_f\alpha_{K_f} \tag{4}$$

$$F_{K_i} = K_i\alpha_{K_i}^{3/2} \tag{5}$$

where the subscripts f and i refer to the flexion and indentation components respectively.

The $K_f$ and $K_i$ constants for an isotropic material are described by Eqs. (6) and (7).

$$K_f = \frac{4\pi E e^3}{3(1-v)(3+v)a^2} \tag{6}$$

$$K_i = \frac{4\sqrt{R}}{3}\left(\frac{1-v^2}{E} + \frac{1-v_i^2}{E_i}\right)^{-1} \tag{7}$$

where $E$, $e$ and $v$ are the elastic modulus, width, and Poisson coefficient, respectively, $a$ is the support diameter, and $R$, $E_i$ and $v_i$ are the indenter radius, elastic modulus and Poisson coefficient. In this way $C_f$ is described as

$$C_f = \frac{\sqrt{K_f m\left(1 + \left(\frac{\pi}{\ln\varepsilon}\right)^2\right)}}{2} \tag{8}$$

The constant $C_i$ is iteratively calculated until the model fits the restitution coefficient expression $\varepsilon$ of Eq. (9):

$$\varepsilon = \frac{\int_0^{t_c} F\,dt}{mv_0} - 1 \tag{9}$$

The model configuration is shown in Fig. 1a, having four displacements. Due to the serial configuration model, the applied force in each element is the same. Since the dashpots are linear elements, these can be merged (Fig. 1b), and the equivalent dashpot constant can be expressed as

$$C_{eq} = \frac{1}{\frac{1}{C_f} + \frac{1}{C_i}} \tag{10}$$

Then, the system performance can be described by the differential equations:

$$m\ddot{\alpha} = K_f \alpha_1 \tag{11}$$

$$C_{eq}\dot{\alpha}_2 = K_i \alpha_3^{3/2} \tag{12}$$

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3 \tag{13}$$

Reducing variables, from (11)–(13) the expressions (14) and (15) can be obtained:

$$\ddot{\alpha} = \frac{K_f}{m}\left(\alpha - \alpha_2 - \left[\frac{C_{eq}}{K_i}\dot{\alpha}_2\right]^{2/3}\right) \tag{14}$$

$$\dot{\alpha}_2 = \frac{K_f}{C_{eq}}\left(\alpha - \alpha_2 - \left[\frac{C_{eq}}{K_i}\dot{\alpha}_2\right]^{2/3}\right) \tag{15}$$

## 5. Numeric solution

Since Eqs. (14) and (15) do not have an analytical solution, a Runge–Kutta fourth order (Curtis, 1987) numerical method can be used to solve the system. Then, two more expressions are necessary to calculate the variables. Defining:

$$\dot{\alpha} = \frac{d\alpha}{dt} \tag{16}$$

Since the $\dot{\alpha}_2$ cannot be calculated straight forward an initial value of cero can be input in the approximation of Eq. (17):

$$\dot{\alpha}_2 = \frac{d\alpha_2}{dt} = \frac{\alpha_{2(i)} - \alpha_{2(i-1)}}{\Delta t} = \frac{\Delta\alpha_2}{\Delta t} \tag{17}$$

The formulae to program the algorithm are summarized by the equations system:

$$f_1 = \frac{K_f}{m}\left(\alpha - \alpha_2 - \left[\frac{C_{eq}}{K_i}\dot{\alpha}_2\right]^{2/3}\right) \tag{18}$$

$$f_2 = \frac{K_f}{C_{eq}}\left(\alpha - \alpha_2 - \left[\frac{C_{eq}}{K_i}\dot{\alpha}_2\right]^{2/3}\right) \tag{19}$$

$$f_3 = \dot{\alpha} \tag{20}$$

The method was programmed in a worksheet with a 1.5 μs interval, time similar to the experimental resolution of the used equipment, taking as start values the impact velocity ($\dot{\alpha}_{t=0} = v_0$) and the initial position of all the elements ($\alpha_1 = \alpha = \alpha_2 = \alpha_3 = 0$), being the ram mass ($m$), and the constants $K_i$, $K_f$ and $C_{eq}$ the models parameters. The value of the restitution coefficient in Eq. (8) comes from the application of Eq. (9) over the experimental $F$–$t$ curves. Using a finite number of iterations, the restitution coefficient of the numerical curve reaches the experimental coefficient using the $C_i$ value.

## 6. Results

Fig. 2 shows a good fit of the analytical model over an experimental curve from an impact test on a HIPS + PS + glass-beads sample with a fall of 40 mm, similar to the other materials. However, at greater impact velocities, a separation between the experimental and the numerical curve is shown in the recovery phase of samples that show cracks after the test (Fig. 3). These cracks are supposed to change the sample compliance $K_f$ due to the presence of a non continuum space inside the material after the maximum displacement, and then, the pseudo frequency or damping natural frequency of the sample is changed, having a larger period. This is more evident in materials like the HIPS.

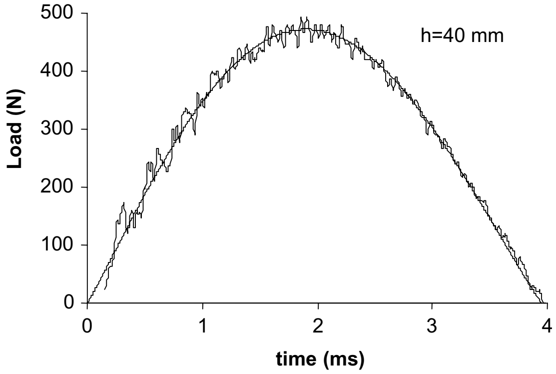


Fig. 2. Model fits over an experimental curve from an impact test on HIPS + PS + μeV sample with 40 mm fall.

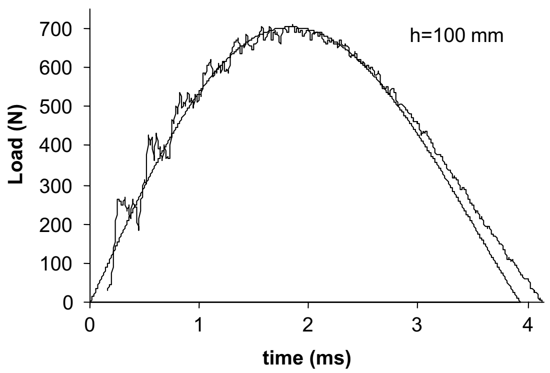


Fig. 3. Model fits over an experimental curve from an impact test on HIPS + PS + μeV sample with 100 mm fall.

When the curve fitting is applied to the experimental $F$–$t$ curve, it is possible to acquire the material parameters. Table 1 shows the variation of the elastic modulus (average) versus the material composition. The elastic modulus has a lower value as the HIPS fraction increases. When a rigid phase is present a little increase is shown. The values obtained are coherent with previous results, and follow the mixture rule.

Fig. 4 shows the HIPS elastic modulus and the restitution coefficient evolution versus the impact velocity. In a similar way, all the materials show a little increase of the elastic modulus and a decrease of the restitution coefficient when the impact velocity is increased. This can be explained due the fact that a greater velocity the viscoelastic phenomena will be less important like a deformation mechanism, while other non elastic mechanisms of deformation, like craze generation, increase their presence.

In fact, the $C_i$ and $C_f$ constants decrease like a consequence of this increment of a non elastic and non viscoelastic behavior, being the $C_f$ lower, as a sign that the most important energy losses are consequence of the flexion component.

The elastic modulus values obtained through the method are lower than those obtained in the indentation impact test realized by Jimenez et al. (2002), but greater than the results of the pure flexion impact

Table 1
Material parameters average value according to the model

| Material | $E$ (Gpa) | $t_c$ (ms) | $V_{0\ max}$ (m/s) |
|---|---|---|---|
| PS | 3.776 | 3.908 | 0.95 |
| 25% HIPS | 3.595 | 4.050 | 1.68 |
| 50% HIPS | 3.371 | 4.193 | 2.26 |
| 75% HIPS | 3.035 | 4.402 | 2.50 |
| HIPS | 2.733 | 4.704 | 2.79 |
| PS-μeV | 4.098 | 3.660 | 1.17 |
| PS-HIPS-μeV | 3.500 | 3.953 | 1.48 |
| HIPS-μeV | 2.998 | 4.478 | 1.48 |

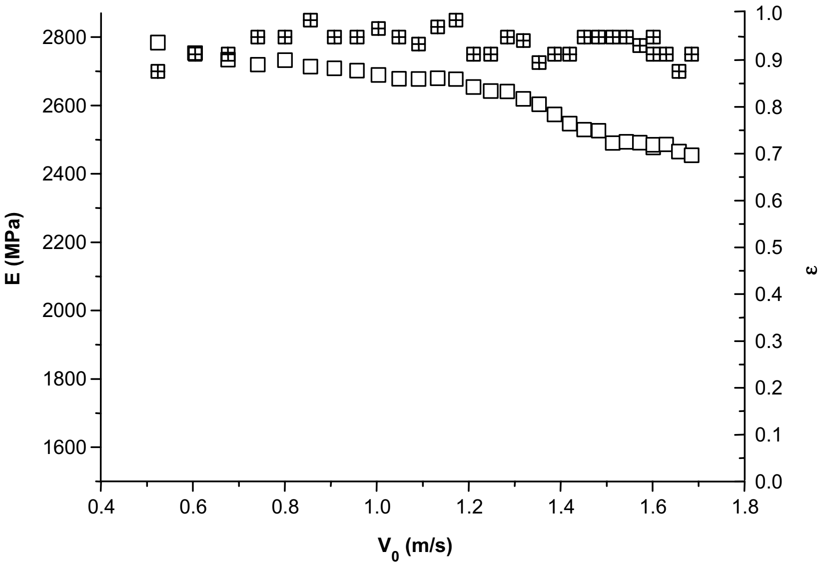


Fig. 4. Values obtained from the model application over the experimental curves of HIPS: ⊞ elastic modulus, □ restitution coefficient.

Table 2
Comparison of the average elastic modulus for different materials according to the model proposed and Martinez et al. (2001) and Jimenez et al. (2002) models

| Material | Flexion model | Composed model | Indentation model |
|---|---|---|---|
| HIPS | 2.51 | 2.73 | 2.95 |
| 25% PS | 2.64 | 3.03 | 3.13 |
| 50% PS | 3.11 | 3.37 | 3.45 |
| 75% PS | 3.43 | 3.6 | 3.66 |
| PS | 3.41 | 3.78 | 3.78 |
| HIPS + μeV | 2.61 | 3 | 3.1 |
| PS + HIPS + μeV | 3.15 | 3.5 | 3.55 |
| PS + μeV | 3.64 | 4.1 | 4.12 |

model (Jimenez et al., 2002; Martinez et al., 2001; Sullcahuaman, 2001) like shown in the Table 2. The differences between the results are consequence of the differences in the strain rate of the flexion and impact tests, and the inclusion of the indentation phenomena in the flexion impact model. In this way, the evaluation of the material behavior is corrected.

## 7. Conclusions

The application of the proposed non linear and non conservative model is successful to evaluate the material, and represents their behavior in the velocity range. With the fit of the model it is possible to determine the energy level, when the damages in the sample start, and the restitution coefficient and elastic modulus at high loading rates.

## References


Casiraghi, T. et al., 1988. Fracture mechanics of polymers. Critical evaluation of linear elastic behaviour at high speed testing. Journal of Material Science 23, 459–466.

Curtis, G., 1987. Análisis Numérico, second ed. Representaciones y servicios de ingeniería, México.

Greszczuk, L.B., 1982. Damage in composite materials due to low velocity impact. In: Impact Dynamics. John Wiley & Sons, New York, pp. 55–94.

Jimenez, O. et al., 2002. Indentación por impacto de baja energía, Anales de mecánica de la fractura, X Reunión del grupo español de Fractura, Girona, Spain.

Martinez, A.B. et al., 1994. Impacto de Plásticos: Impacto de baja energía. Información tecnológica, 19–24.

Martinez, A.B., et al., 2001. Impacto de baja energía en composites de poliestireno. IV Congreso Nacional de Materiales Compuestos, Gijón, Spain.

Pang, S.S., 1991. Impact response of composite laminates whit hemispherical indenter. Polymer Engineering and Science 31, 1461–1466.

Rayleigh, L., 1906. On the Production of vibration by forces of relatively long duration with the applications to the theory of collisions. Philosophical Magazine 11 (3), 283.

Sullcahuaman, A., 2001. Fractura de materiales poliméricos a altas velocidades de solicitación, Tesis doctoral, Universitat Politécnicade Catalunya. Barcelona, Spain.